Slab – Column Frames

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Presentation Overview

- **Current Practice**
  - Modeling & analysis
  - Connection design
  - Progressive collapse
  - Deformation compatibility

- **Existing Construction**
  - Post-earthquake observations
  - Modeling and Model Assessment
  - Backbone curves/Rehabilitation
Current Practice

- Non-participating or “gravity” system
- Post-tensioned slab-column frame
- Span-to-depth ratios typically ~40+
- Use of shear reinforcement at slab-column connection to allow for thinner slabs or to eliminate drop panels

~1/3 scale shake table test specimen
Shear Reinforcement

Gravity Load Analysis & Design

- ACI 318 Chapter 11, 13, & 21 Materials
  - Slab moments: Use direct design, Equivalent frame, or computer program
  - Connection design - Chapter 11 & 13

\[ w_u = 1.2D + 1.6L \]

\[ EI_{\text{column}} \]

\[ EI_{\text{slab}} = E_c(\alpha \beta I_2) \]

Effective slab width
Gravity Load Analysis - Moments

Gravity Analysis: 1.2D + 1.6L

- Design slab-column connection to transfer unbalanced moment to column
- FEMA 356 refers to ACI 318 provisions
Unbalanced Moment Transfer

Unbalanced moment at the slab-column connection is transferred by two mechanisms:

- Moment transfer (flexure) over a transfer width of \( c + 3h \) centered on the column
- Eccentric shear on a critical section around the slab-column connection

Code provisions are covered in Chapter 13 (13.5) and Chapter 11 (11.12) of ACI 318

\[
M_f = \gamma_f M_{\text{unbalanced}}
\]

where \( \gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \)

\( b_1, b_2 = \) widths of critical section defined in 11.12.1.2

\[
M_v = (1 - \gamma_f)M_{\text{unbalanced}} = \gamma_v M_{\text{unbalanced}}
\]

If \( b_1 = b_2 \), then:

\[
\gamma_f = 0.6 \quad \text{and} \quad \gamma_v = 0.4
\]
Unbalanced Moment Transfer

Unbalanced Moment (Interior connection)

Flexural Transfer: \( c_2 + 3h \)
- \( \gamma_f M_{unb} \) where \( \gamma_f \) is typically \( \sim 0.6 \) for square columns
- Ratio of top to bottom reinforcement of 2:1 recommended in ACI 318 (R13.5.3.3)

\[ M_{unb} = M_L + M_R \]

- FEMA 356 6.5.4.3(2) allows use of \( c_2 + 5h \)
Unbalanced Moment Transfer

**Eccentric Shear transfer**

- Critical section is defined $d/2$ from column face
- Direct shear stress
  - $b_0 = \text{perimeter of critical section}$
    \[
    v_{\text{gravity}} = \frac{V_{u(\text{direct})}}{b_0 d}
    \]
- Eccentric shear stress due to
  \[
  (1-\gamma_f) M_{\text{unb}} = \gamma_v M_{\text{unb}}
  \]
  \[
  v_{\text{unb}} = \gamma_v \frac{M_{\text{unb}} z}{J}
  \]
Unbalanced Moment Transfer

- Combined shear stresses
- Check punching failure per 318

\[ \phi v_n = \phi v_c \geq v_u \quad \text{where } \phi = 0.75 \]

\[ v_c = \min \left\{ \sqrt{f'_c} \left( 2 + \frac{4}{\beta_c} \right) \right\} \]

\[ = 2\sqrt{f'_c} \left( 2 + \frac{\alpha_s + d}{b_0} \right) \]

\[ = 4\sqrt{f'_c} \]

ACI Eq. 11-33, -34, -35

Direct shear stress

Eccentric shear stress

Total shear stress

\[ v_{u,\max} \leq \phi v_n \]
Laboratory Studies

Progressive collapse - continuous bottom steel (2 bars)
ACI 318-05 7.13.2.5 (13.3.8.5)

ACI Committee 352.1R89
Slab – Column Report

Fig. 5.4-Model of connection during punching failure

\[ A_{sm} = 0.5 \frac{w_u l_1 l_2}{\phi f_y} \]

Recommendations for the design of slab-column connections in monolithic RC Structures, ACI-ASCE Committee 352, Report 352.1R-89 (reapproved 1997)
Deformation Compatibility

- Slab – column (gravity) frame assessment

  - Included in the model with the lateral system

  \[ w_u = 1.2D + 0.5L \]

Imposed lateral displacements (new design)

Pushover Analysis (Assessment of existing)

\[ EI_{column} \]

\[ EI_{slab} = E_c(\alpha \beta I_2) \]
Deformation Compatibility

- Determine if the connection can resist the $V_u$ & $M_{unb}$ without punching failure – Adequate strength. (ACI 318-05 21.11.5)
  - Flexural transfer, eccentric shear stress model
  - Limit analysis approach – for connections with a fuse
  - this does not consider the potential for shear strength degradation.
Alternative - Deformation Compatibility

- Verify that punching failures do not occur for gravity shear combined with imposed interstory displacement for $A_M$ (new) or $\delta_{\text{target}}$ (Rehab). Adequate deformability. (ACI 318-05 21.11.5)

$RC$ interior and exterior (limited data) connections

$\frac{V_g}{\phi V_c}$, where $V_c = \left(\frac{1}{3}\right)f'_c \frac{1}{2} b_o d$

Isolated RC "Interior" Connections $^2$,$^3$,$^4$,$^5$,$^6$,$^7$,$^8$,$^9$,$^10$, $^{11}$,$^{12}$,$^{13}$,$^{14}$,$^{15}$,$^{16}$,$^{17}$

Subassemblies $^6$

Nine-panel Frame $^8$

Isolated RC "Edge" Connections $^9$

Relationship for RC with stud-rails (Robertson et al. $^{10}$)

Best-Fit Line for Interior Connections without Shear Reinforcement

ACI 318-05 Limit
Deformation Compatibility

- PT Connections without shear reinforcement

Gravity Shear Ratio ($\frac{V_g}{\phi V_c}$), where $V_c = (0.29f'_{c}^{1/2} + 0.3f_{pc})b_od$
Presentation Overview

- Current Practice
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  - Deformation compatibility

- Existing Construction
  - Background & observations
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  - Backbone curves/Rehabilitation
Older Construction

- Gravity design, or relatively low lateral forces used for design

- Bent reinforcement sometimes used

- No continuous bottom reinforcement through column cage
Post-Earthquake Observations

Bullock’s Department Store - Northridge Fashion Mall
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Modeling Overview

- How to model...
  - Lateral stiffness?
  - Connection behavior?
- How good are our models?
  - Shake table studies
- FEMA 356 backbone curves
  - Basis of existing curves
  - New information?
Modeling Assumptions - Typical

\[ EI_{\text{eff}} = \text{effective column stiffness} \]

\[ EI_{\text{eff}} = \alpha \beta E I_g \]

Rigid end zones at joints

Model slab with “an effective beam”
Analytical Model - Column Stiffness

- $P_G =$ axial from gravity and $P_E =$ axial from earthquake
- Anchorage slip - not likely as significant as noted for beam-column frames (see Elwood presentation)
**Analytical Model – Slab Flexural Stiffness**

**Effective Beam Width Model**

\[ \alpha \]: Effective Beam Width Factor

\[ \beta \]: Coefficient accounting for Cracking

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**Kang & Wallace (2005)**

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>PT</th>
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</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.75</td>
<td>0.65</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.33</td>
<td>0.5</td>
</tr>
</tbody>
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Analytical Modeling - Connections

M<sub>unbalanced</sub> @connection
Flexure c<sub>2</sub>+3h (5h)
Eccentric shear
M<sub>n</sub> = M<sub>f</sub>/0.6
M<sup>−</sup><sub>n</sub> / M<sup>+</sup><sub>n</sub> @ column strip

Punching before or after yielding

Limit State Model: Mean – 1 σ

This model satisfies FEMA 356 6.5.4.2.2, which states that the connection must be modeled separately from slab and column elements.
Connection Modeling - Punching

Gravity Shear Ratio ($V_g / \phi V_c$), where $V_c = (1/3)f_c^{1/2}b_o d$

- Isolated RC "Interior" Connections
- Subassemblies
- Nine-panel Frame
- Isolated RC "Edge" Connections

Best-Fit Line for Interior Connections without Shear Reinforcement

ACI 318-05 Limit

Relationship for RC with stud-rails (Robertson et al.)
Shake Table Studies

- Two stories, $2 \times 2$ bays
- Approximately $1/3$ scale

RC Specimen

- Six 200 x 200 mm columns
- 90 mm thick slab
- 9.5 mm rebar $f_y = 414$ MPa
- $f'_c = 28$ MPa

Dimensions:
- 4.1 m
- 4.3 m
RC Specimen - Reinforcement

Interior Connection

Shear Reinforcement

Expected connection behavior: Flexural yielding, followed by punching
PT Specimen

5.7 m

8 mm 7-wire strand

6.35 mm deformed rebar
PT Specimen – Interior

ACI318-05 Requires only bottom (integrity) reinforcement
PT Video – Run 5
Model Assessment - NSP

Top Drift [%]

Base Shear [kN]

Top displacement relative to footing [mm]

RC-RUN4-Exp
Push-over (2:1)
Push-over (1:2)

θ_u = 2.5%

(1:2 Ratio)
(2:1 Ratio)
Model Assessment - PT

See Kang et al., 13WCEE, August 2004, paper 1119
Direct measurement of footing rotations
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- **Shear reinforcement**
Deformation – Backbone Curves

<table>
<thead>
<tr>
<th>Slabs Controlled by Flexure</th>
<th>Model Parameters, Radians</th>
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<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{V_{\text{gravity}}}{V_0}$</td>
<td>Continuity Reinforcement</td>
</tr>
<tr>
<td>≤ 0.2</td>
<td>Yes</td>
</tr>
<tr>
<td>&gt; 0.4</td>
<td>Yes</td>
</tr>
<tr>
<td>≤ 0.2</td>
<td>No</td>
</tr>
<tr>
<td>≥ 0.25</td>
<td>No</td>
</tr>
</tbody>
</table>

$V_u = 1.2D + 0.5L$

Continuity reinforcement defined as at least one bottom bar or pt bar continuous through the column cage in each direction.
Slab – Column Tests

Typical test setup

- Cyclic lateral load
- Gravity load
- Axial load
- Reaction block
- Strong Floor

\[ \delta_{\text{column}} \]

\[ \delta_{\text{slab}} \]
New Data - Test Results #1

- Slab reinforcing details

10ft x 10 ft x 4.5”

2 Continuous bottom bars

Figure 2-Slab Reinforcement for ND1C, ND4LL, and ND5XL.
Test Results #1 – Control Specimen

\[ \frac{V_g}{V_c} = 0.23 \]

\[ M_n = (12)(71 \text{ mm}^2)(414 \text{ MPa})(95\text{ mm}) = 33,500 \text{ kN-mm} \]
\[ P_n = 33,500 \text{ kN-mm}/1524\text{ mm} = 22 \text{ kN} \]
\[ \theta_e = 0.015; \quad \theta_p = 0.02(17/20) = 0.017 \]
Test Results #1

\[
\frac{V_g}{V_c} = 0.28
\]

\[M_n = (12)(71 \text{ mm}^2)(414 \text{ MPa})(95\text{mm}) = 33,500 \text{ kN-mm}\]

\[P_n = 33,500 \text{ kN-mm} / 1524\text{mm} = 22 \text{ kN}\]

\[\theta_c = 0.015; \quad \theta_p = 0.02(12 \div 20) = 0.012\]
Test Results #1

\[ \frac{V_g}{V_c} = 0.48 \]

\[ M_n = (12)(71 \text{ mm}^3)(414 \text{ MPa})(95\text{mm}) = 33,500 \text{ kN-mm} \]

\[ P_n = \frac{33,500 \text{ kN-mm}}{1524 \text{ mm}} = 22 \text{ kN} \]

\[ \frac{\theta_c}{\theta_p} = 0.015; \quad \theta_p = 0 \]
Test Results #1 - Summary

- FEMA 356 – Overall comparison
Test Results - #2

- Slab reinforcing details – less reinforcement

**Figure 4-Slab Reinforcement for ND7LR.**

10ft x 10 ft x 4.5”

2 Continuous bottom bars
\[ \frac{V_g}{V_c} = 0.36 \]

\[ M_n = (7)(71 \text{ mm}^2)(414 \text{ MPa})(95\text{ mm}) = 19,500 \text{ kN-mm} \]

\[ P_n = 19,500 \text{ kN-mm/1524mm} = 13 \text{ kN} \]

\[ \theta_e = 0.015; \quad \theta_p = 0.02(4/20) = 0.004 \]
Test Results - Summary

Backbone relation:
P = 10 kip (arbitrary)
\( \theta_e \approx 0.01 \quad \theta_p = 0.02 \)

Straight bars vs Bent up bars
- no difference
- except for collapse

Fig. 5—Envelopes of load-drift hysteresis loops for subassemblies

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- **Shear reinforcement**
Summary

● Modeling
  ■ Effective beam width model
  ■ Connection behavior – Limit state model

● Backbone curves - RC
  ■ Conservative – In general
  ■ Review allowable plastic rotation for low gravity stress ratios < 0.2, mean - $\sigma$
  ■ Potential to increase plastic rotation for low reinforcement ratios
  ■ Remove residual capacity for RC connections
Summary

Backbone curves - PT

- Conservative
- Increase plastic rotation from 0.02 (RC) to 0.03 at gravity shear ratio of 0.2
- Review higher gravity shear ratios – allowable plastic rotation of 0.01 at a gravity shear ratio of 0.5
- Allow residual capacity of 0.2 up to drifts of about 5% where one strand pass within the column cage in both directions.
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